

Evaluate :

$$\int_{-1}^2 |x^3 - x| dx$$

CBSE 2012

Solution

$$\int_{-1}^2 |x^3 - x| dx = I = \int_{-1}^1 |x^3 - x| dx + \int_1^2 |x^3 - x| dx$$

$$\int_{-1}^1 |x^3 - x| dx = 2 \int_0^1 |x^3 - x| dx$$

$$I = 2 \int_0^1 |x^3 - x| dx + \int_1^2 |x^3 - x| dx$$

$$x > x^3$$

$$x^3 > x$$

$$I = 2 \int_0^1 (x - x^3) dx + \int_1^2 (x^3 - x) dx$$

$$= 2 \left(\frac{x^2}{2} - \frac{x^4}{4} \right) \Big|_0^1 + \left(\frac{x^4}{4} - \frac{x^2}{2} \right) \Big|_1^2$$

$$= 2 \left[\left(\frac{1}{2} - \frac{1}{4} \right) - 0 \right] + \left[\left(\frac{16}{4} - \frac{4}{2} \right) - \left(\frac{1}{4} - \frac{1}{2} \right) \right]$$

$$= 3 \left(\frac{1}{2} - \frac{1}{4} \right) + (4 - 2)$$

$$= \frac{3}{4} + 2 = \frac{11}{4}$$