Find $S_\infty = 1 + \frac{1+2}{2!} + \frac{1+2+3}{3!} + \frac{1+2+3+4}{4!} + ...$upto infinite terms.

Solution

$$S_\infty = \sum \frac{1+2+3+4...........+r}{r!}$$

$$= \sum \frac{r(r+1)}{2r!}$$

$$= \sum \frac{(r+1)}{2(r-1)!}$$

$$= \sum \frac{[(r-1)+2]}{2(r-1)!}$$

$$= \sum \frac{1}{2(r-2)!} + \frac{1}{(r-1)!}$$

$$= \sum \frac{1}{2(r-2)!} + \sum \frac{1}{(r-1)!}$$

$$= \sum_{r=2}^{r=\infty} \frac{1}{2(r-2)!} \quad (r \geq 2 \text{ since negative factorial is not defined}) + \sum_{r=1}^{r=\infty} \frac{1}{(r-1)!}$$

$$= \frac{e}{2} + e$$