

Prove that,  $\frac{\cot \theta + \cos ec\theta - 1}{\cot \theta - \cos ec\theta + 1} = \frac{1 + \cos \theta}{\sin \theta}$

$$\begin{aligned} \text{LHS} &= \frac{\cot \theta + \cos ec\theta - 1}{\cot \theta - \cos ec\theta + 1} \\ &= \frac{\cot \theta + \cos ec\theta - 1}{\frac{(\cot \theta - \cos ec\theta)}{(\cot \theta + \cos ec\theta)} \times (\cot \theta + \cos ec\theta) + 1} \\ &= \frac{\cot \theta + \cos ec\theta - 1}{\frac{\cot^2 \theta - \cos ec^2 \theta}{\cot \theta + \cos ec\theta} + 1} \\ &= \frac{\cot \theta + \cos ec\theta - 1}{\frac{-1}{\cot \theta + \cos ec\theta} + 1} \\ &= \frac{(\cot \theta + \cos ec\theta - 1)(\cot \theta + \cos ec\theta)}{\cot \theta + \cos ec\theta - 1} \\ &= \cot \theta + \cos ec\theta \\ &= \frac{\cos \theta + 1}{\sin \theta} = \text{RHS} \end{aligned}$$