A bullet is fired vertically upwards with velocity υ from the surface of a spherical planet. When it reaches its maximum height, its acceleration due to the planet's gravity is 1/4th of its value at the surface of the planet. If the escape velocity from the planet is $\upsilon_{esc} = \upsilon\sqrt{N}$, then the value of N is (ignore energy loss due to atmosphere)

Solution

Using conservation of mechanical energy,

$$-\frac{GMm}{R}+\frac{1}{2}mv^2=-\frac{GMm}{R+h}$$

$$\therefore -\frac{g_0R^2}{R}+\frac{1}{2}v^2=-\frac{g(R+h)^2}{R+h}$$

$$\therefore -g_0R+\frac{1}{2}v^2=-g(R+h) \quad ....(*)$$

Now, $g=\dfrac{g_0}{\left(1+\dfrac{h}{R}\right)^2}$

$$\therefore \frac{g_0}{4}=\frac{g_0}{\left(1+\dfrac{h}{R}\right)^2}$$

$$\therefore h=R$$

$$\therefore -g_0R+\frac{1}{2}v^2=-\frac{g_0}{4}(R+R)=-\frac{1}{2}g_0R \qquad \text{(From *)}$$

$$\therefore v^2=g_0R$$

Now, $v_{esc}=\sqrt{2g_0R}=v\sqrt{N}$

$$\therefore \sqrt{2v^2}=v\sqrt{N}$$

$$\therefore N=2$$