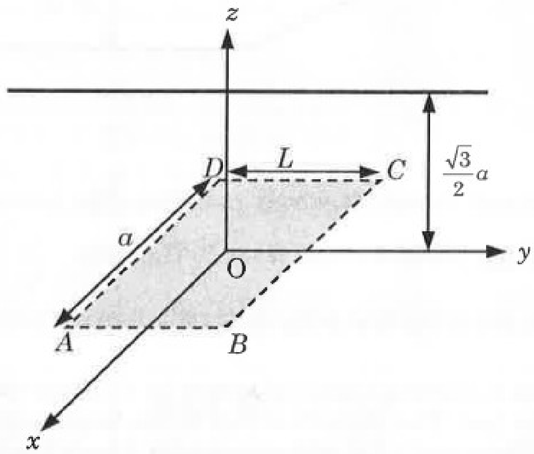
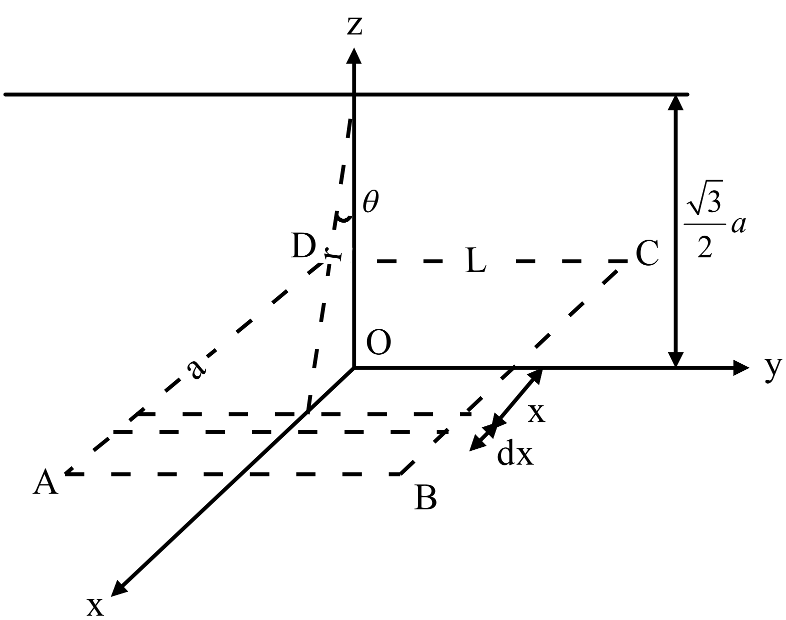
An infinitely long uniform line charge distribution of charge per unit length λ lies parallel to the y-axis in the y-z plane at $z = \frac{\sqrt{3}}{2}a$ (see figure). If the magnitude of the flux of the electric field through the rectangular surface $ABCD$ lying in the x-y plane with its centre at the origin is $\frac{\lambda L}{n\varepsilon_0}$ (ε_0 = permittivity of free space), then the value of n is



Solution



On the surface ABCD, consider a thin strip of thickness dx at x,

Flux through it $= d\phi = \frac{\lambda}{2\pi\epsilon_0 r}.Ldx.\cos\theta$

$$\therefore d\phi = \frac{\lambda}{2\pi\epsilon_0 r}.Ldx.\frac{\frac{\sqrt{3}}{2}a}{r} = \frac{\sqrt{3}\lambda La}{4\pi\epsilon_0\left\{x^2+\left(\frac{\sqrt{3}}{2}a\right)^2\right\}}dx$$

$$\therefore \phi = \int_{-\frac{a}{2}}^{\frac{a}{2}} \frac{\sqrt{3}\lambda La}{4\pi\epsilon_0\left\{x^2+\left(\frac{\sqrt{3}}{2}a\right)^2\right\}}dx = \frac{4\sqrt{3}\lambda La}{4\pi\epsilon_0}\int_{-\frac{a}{2}}^{\frac{a}{2}}\frac{dx}{4x^2+3a^2}$$

$$\therefore \phi = 2\times\frac{\sqrt{3}\lambda La}{\pi\epsilon_0}\int_0^{\frac{a}{2}}\frac{dx}{4x^2+3a^2}$$

$$\therefore \phi = \frac{2\sqrt{3}\lambda La}{\pi\epsilon_0}\times\frac{1}{a\sqrt{3}}\times\frac{1}{2}\times\tan^{-1}\frac{2x}{a\sqrt{3}}\Bigg|_0^{\frac{a}{2}}$$

$$\therefore \phi = \frac{\lambda L}{\pi\epsilon_0}\times\frac{\pi}{6} = \frac{\lambda L}{\epsilon_0}\times\frac{1}{6} = \frac{\lambda L}{n\epsilon_0}$$

$$\therefore n = 6$$