Which of the following values of α satisfy the equation

$$\begin{vmatrix} (1+\alpha)^2 & (1+2\alpha)^2 & (1+3\alpha)^2 \\ (2+\alpha)^2 & (2+2\alpha)^2 & (2+3\alpha)^2 \\ (3+\alpha)^2 & (3+2\alpha)^2 & (3+3\alpha)^2 \end{vmatrix} = -648\alpha \ ?$$

(A) -4 (B) 9 (C) -9 (D) 4

Solution

Applying, $R_3 \to R_3 - R_2$ & $R_2 \to R_2 - R_1$

$$\begin{vmatrix} (1+\alpha)^2 & (1+2\alpha)^2 & (1+3\alpha)^2 \\ 3+2\alpha & 3+4\alpha & 3+6\alpha \\ 5+2\alpha & 5+4\alpha & 5+6\alpha \end{vmatrix} = -648\alpha$$

Applying, $R_3 \to R_3 - R_2$

$$\begin{vmatrix} (1+\alpha)^2 & (1+2\alpha)^2 & (1+3\alpha)^2 \\ 3+2\alpha & 3+4\alpha & 3+6\alpha \\ 2 & 2 & 2 \end{vmatrix} = -648\alpha$$

Applying, $C_3 \to C_3 - C_2$ & $C_2 \to C_2 - C_1$

$$\begin{vmatrix} (1+\alpha)^2 & (2+3\alpha)\alpha & (2+5\alpha)\alpha \\ 3+2\alpha & 2\alpha & 2\alpha \\ 2 & 0 & 0 \end{vmatrix} = -648\alpha$$

$$\Rightarrow \alpha^2 \begin{vmatrix} (1+\alpha)^2 & 2+3\alpha & 2+5\alpha \\ 3+2\alpha & 2 & 2 \\ 2 & 0 & 0 \end{vmatrix} = -648\alpha$$

Expanding the determinant about R_3,

$\alpha^2.2\{2(2+3\alpha)-2(2+5\alpha)\} = -648\alpha$

$\therefore \alpha^2.\alpha = 81\alpha$ or $\alpha(\alpha^2 - 81) = 0$

$\therefore \alpha = -9, 0, 9$

Hence, (B) & (C).