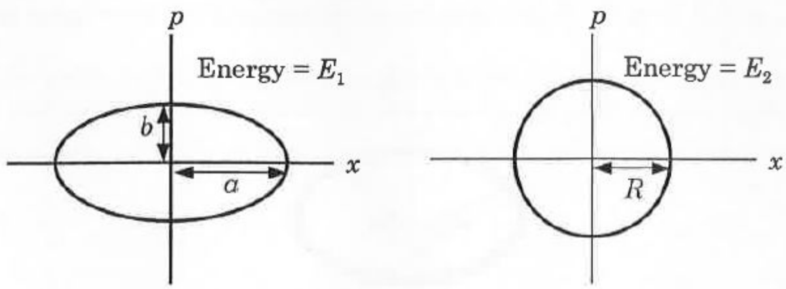
Two independent harmonic oscillators of equal mass are oscillating about the origin with angular frequencies ω_1 and ω_2 and have total energies E_1 and E_2, respectively. The variations of their momenta p with positions x are shown in the figures. If $\frac{a}{b} = n^2$ and $\frac{a}{R} = n$, then the correct equation(s) is(are)



(A) $E_1\omega_1 = E_2\omega_2$ (B) $\frac{\omega_2}{\omega_1} = n^2$ (C) $\omega_1\omega_2 = n^2$ (D) $\frac{E_1}{\omega_1} = \frac{E_2}{\omega_2}$

Solution

$$E_1 = \frac{1}{2}m\omega_1^2 a^2 \quad \& \quad E_2 = \frac{1}{2}m\omega_2^2 R^2$$

$$\therefore \frac{E_1}{E_2} = \frac{\omega_1^2 a^2}{\omega_2^2 R^2} = \frac{\omega_1^2}{\omega_2^2}.n^2 \quad \text{.....................(*)}$$

$$v_0 = \omega A \quad \text{or} \quad p_0 = m\omega A$$

$\therefore b = m\omega_1 a \quad \& \quad R = m\omega_2 R$ (assuming that the second graph is a circle of radius R)

$$\therefore \omega_1 = \frac{1}{m}.\frac{b}{a} \quad \& \quad \omega_2 = \frac{1}{m}$$

$$\therefore \omega_1 = \omega_2.\frac{b}{a} = \omega_2.\frac{1}{n^2}$$

$\therefore \frac{\omega_2}{\omega_1} = n^2$ {Option (B) is correct}

Also from (*), $\frac{E_1}{E_2} = \frac{\omega_1^2}{\omega_2^2}.n^2 = \frac{\omega_1^2}{\omega_2^2}.\frac{\omega_2}{\omega_1} = \frac{\omega_1}{\omega_2}$

$\therefore \frac{E_1}{\omega_1} = \frac{E_2}{\omega_2}$ {Option (D) is also correct}

Hence, (B) & (D).