In $\mathbb{R}^3$, consider the planes $P_1 : y = 0$ and $P_2 : x + z = 1$. Let P_3 be a plane, different from P_1 and P_2, which passes through the intersection of P_1 and P_2. If the distance of the point $(0, 1, 0)$ from P_3 is 1 and the distance of a point (α, β, γ) from P_3 is 2, then which of the following relations is (are) true?

(A) $2\alpha + \beta + 2\gamma + 2 = 0$

(B) $2\alpha - \beta + 2\gamma + 4 = 0$

(C) $2\alpha + \beta - 2\gamma - 10 = 0$

(D) $2\alpha - \beta + 2\gamma - 8 = 0$

Solution

$P_3 \equiv (x + z - 1) + \lambda(y) = 0$

Or $P_3 \equiv x + \lambda y + z - 1 = 0$

Distance of above plane from (0, 1, 0) = $\left| \dfrac{0 + \lambda + 0 - 1}{\sqrt{1^2 + \lambda^2 + 1^2}} \right| = 1$

$\therefore \left| \dfrac{\lambda - 1}{\sqrt{\lambda^2 + 2}} \right| = 1$

$\therefore \lambda^2 - 2\lambda + 1 = \lambda^2 + 2$

$\therefore \lambda = -\dfrac{1}{2}$

Now, distance of P_3 from $(\alpha, \beta, \gamma) = \left| \dfrac{\alpha + \lambda.\beta + \gamma - 1}{\sqrt{1^2 + \lambda^2 + 1^2}} \right| = 2$

$\therefore \left| \dfrac{\alpha + \left(-\frac{1}{2}\right).\beta + \gamma - 1}{\sqrt{2 + -\left(\frac{1}{2}\right)^2}} \right| = 2$

$\therefore \left| \dfrac{2\alpha - \beta + 2\gamma - 2}{3} \right| = 2$

$\therefore 2\alpha - \beta + 2\gamma - 2 = \pm 6$

$\therefore 2\alpha - \beta + 2\gamma - 8 = 0$ Or $2\alpha - \beta + 2\gamma + 4 = 0$

Hence, (B) & (D).