Let ΔPQR be a triangle. Let $\vec{a}=\overrightarrow{QR}$, $\vec{b}=\overrightarrow{RP}$ and $\vec{c}=\overrightarrow{PQ}$. If $|\vec{a}|=12$, $|\vec{b}|=4\sqrt{3}$ and $\vec{b}.\vec{c}=24$, then which of the following is (are) true?

(A) $\frac{|\vec{c}|^2}{2}-|\vec{a}|=12$ (B) $\frac{|\vec{c}|^2}{2}+|\vec{a}|=30$

(C) $|\vec{a}\times\vec{b}+\vec{c}\times\vec{a}|=48\sqrt{3}$ (D) $\vec{a}.\vec{b}=-72$

Solution

We have, $\vec{a}+\vec{b}+\vec{c}=\vec{0}$

$\therefore \vec{b}+\vec{c}=-\vec{a}$

$\therefore (\vec{b}+\vec{c})^2=a^2$

$\therefore b^2+c^2+2\vec{b}.\vec{c}=a^2$

$\therefore 48+c^2+48=144$

$\therefore c^2=48$, Option (A) is correct.

Again, $\vec{a}+\vec{b}+\vec{c}=\vec{0}$

$\therefore \vec{a}.\vec{b}+b^2+\vec{c}.\vec{b}=0$

$\therefore \vec{a}.\vec{b}+48+24=0$

$\therefore \vec{a}.\vec{b}=-72$, Option (D) is correct.

Further, $\vec{a}+\vec{b}+\vec{c}=\vec{0}$

$\therefore \vec{a}\times\vec{a}+\vec{a}\times\vec{b}+\vec{a}\times\vec{c}=\vec{0}$

$\therefore \vec{a}\times\vec{b}=\vec{c}\times\vec{a}$

$$\therefore \left|\vec{a}\times\vec{b}+\vec{c}\times\vec{a}\right|=2\left|\vec{a}\times\vec{b}\right|=2ab\sin\theta=2ab\sqrt{1-\left(\frac{\vec{a}.\vec{b}}{ab}\right)^2}=2\times12\times4\sqrt{3}\times\sqrt{1-\left(\frac{-72}{12\times4\sqrt{3}}\right)^2}$$

$$\therefore \left|\vec{a}\times\vec{b}+\vec{c}\times\vec{a}\right|=2\times12\times4\sqrt{3}\times\sqrt{1-\frac{3}{4}}=12\times4\sqrt{3}=48\sqrt{3}$$, Option (C) is correct.

Hence, (A), (C) & (D).