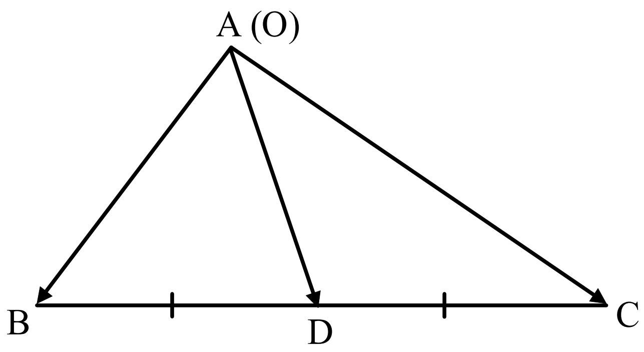
If the vectors $\overrightarrow{AB} = 3\hat{i} + 4\hat{k}$ and $\overrightarrow{AC} = 5\hat{i} - 2\hat{j} + 4\hat{k}$ are the sides of a triangle ABC, then the length of the median through A is:

(1) $\sqrt{18}$ (2) $\sqrt{72}$ (3) $\sqrt{33}$ (4) $\sqrt{45}$

[JEE Main 2013]

Solution



Let observer be placed at A, then

$\overrightarrow{AB}$ = position vector of B & $\overrightarrow{AC}$ = position vector of C

Since D is the mid-point of BC,

position vector of D = $\frac{1}{2}$ (position vector of B + position vector of C)

$$\therefore \overrightarrow{AD} = \frac{1}{2}\left(\overrightarrow{AB} + \overrightarrow{AC}\right) = \frac{1}{2}\left(3\hat{i} + 4\hat{k} + 5\hat{i} - 2\hat{j} + 4\hat{k}\right) = \frac{1}{2}\left(8\hat{i} - 2\hat{j} + 8\hat{k}\right) = 4\hat{i} - \hat{j} + 4\hat{k}$$

$$\therefore AD = \sqrt{4^2 + (-1)^2 + 4^2} = \sqrt{33} unit$$

Hence, (3).