Question

Consider a spherical shell of radius R at temperature T. The black body radiation inside it can be considered as an ideal gas of photons with internal energy per unit volume $u = \frac{U}{V} \propto T^4$ and pressure $p = \frac{1}{3}\left(\frac{U}{V}\right)$. If the shell now undergoes an adiabatic expansion the relation between T and R is:

(1) $T \propto e^{-3R}$ (2) $T \propto \frac{1}{R}$ (3) $T \propto \frac{1}{R^3}$ (4) $T \propto e^{-R}$

Solution

Since, $\frac{U}{V} \propto T^4$ and $p = \frac{1}{3}\left(\frac{U}{V}\right)$, $p \propto T^4$ or $p = KT^4$

For ideal gas, $\frac{pV}{T} = \text{constant}$

$$\therefore \frac{KT^4 \times \frac{4}{3}\pi R^3}{T} = \text{constant}$$

$$\therefore T^3 R^3 \left(\frac{4}{3}\pi K\right) = \text{constant}$$

$$\Rightarrow T^3 R^3 = \text{constant or } TR = \text{constant or } T \propto \frac{1}{R}$$

Hence, Option (2).