A particle of mass m moving in the x direction with speed $2v$ is hit by another particle of mass 2m moving in the y direction with speed v. If the collision is perfectly inelastic, the percentage loss in the energy during the collision is close to :

(1) 50%

(2) 56%

(3) 62%

(4) 44%

Answer: Option (2)

Conservation of momentum in x-direction gives, $2mv = 3mV_x, \therefore V_x = \frac{2}{3}v$

(the two masses stick to each other since collision is perfectly inelastic, hence final mass is 3m)

Conservation of momentum in y-direction gives, $2mv = 3mV_y, \therefore V_y = \frac{2}{3}v$

$$\% \text{ loss in energy} = \frac{K_i - K_f}{K_i} \times 100$$

$$= \frac{\left\{\frac{1}{2}\times m\times(2v)^2 + \frac{1}{2}\times 2m\times v^2\right\} - \left\{\frac{1}{2}\times 3m\times V_x^{\,2} + \frac{1}{2}\times 3m\times V_y^{\,2}\right\}}{\left\{\frac{1}{2}\times m\times(2v)^2 + \frac{1}{2}\times 2m\times v^2\right\}} \times 100$$

$$= \frac{\left\{\frac{1}{2}\times m\times(2v)^2 + \frac{1}{2}\times 2m\times v^2\right\} - \left\{\frac{1}{2}\times 3m\times\left(\frac{2}{3}v\right)^2 + \frac{1}{2}\times 3m\times\left(\frac{2}{3}v\right)^2\right\}}{\left\{\frac{1}{2}\times m\times(2v)^2 + \frac{1}{2}\times 2m\times v^2\right\}} \times 100$$

$$= \frac{(2+1) - \left(\frac{2}{3}+\frac{2}{3}\right)}{(2+1)} \times 100 = \frac{3-\frac{4}{3}}{3} \times 100 = \frac{5}{9} \times 100 \approx 56\%$$