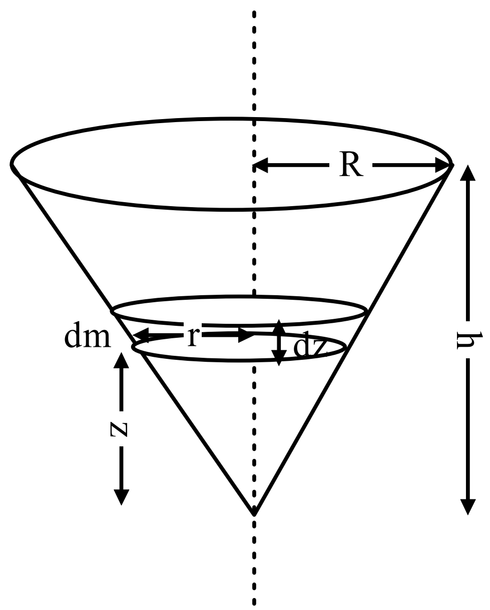
Question

Distance of the centre of mass of a solid uniform cone from its vertex is z_0. If the radius of its base is R and its height is h then z_0 is equal to:

(1) $\frac{3h}{4}$ (2) $\frac{5h}{8}$ (3) $\frac{3h^2}{8R}$ (4) $\frac{h^2}{4R}$

Solution



At a position z from the vertex, consider a thin disc of thickness dz, mass dm and average radius r.

From symmetry, the centre of mass of this thin disc lies on the axis of the cone at a distance z from the vertex.

Let M be the mass of the cone.

$$z_{CM} = z_0 = \frac{\int z\,dm}{M}$$

$$dm = \frac{M}{\frac{1}{3}\pi R^2 h}.\pi r^2 dz = \frac{3Mr^2}{hR^2}dz$$

$$\therefore z_0 = \frac{\int_0^h z\frac{3Mr^2}{hR^2}dz}{M} = \frac{3}{hR^2}\int_0^h zr^2 dz$$

Also, $\frac{r}{R} = \frac{z}{h}$ or $r = \frac{R}{h}z$

$$\therefore z_0 = \frac{3}{hR^2}\int_0^h z\left(\frac{R}{h}\right)^2 z^2 dz = \frac{3}{h^3}\int_0^h z^3 dz = \frac{3}{4}h$$

Hence, Option (1).