Question

From a solid sphere of mass M and radius R a cube of maximum possible volume is cut. Moment of inertia of cube about an axis passing through its center and perpendicular to one of its faces is:

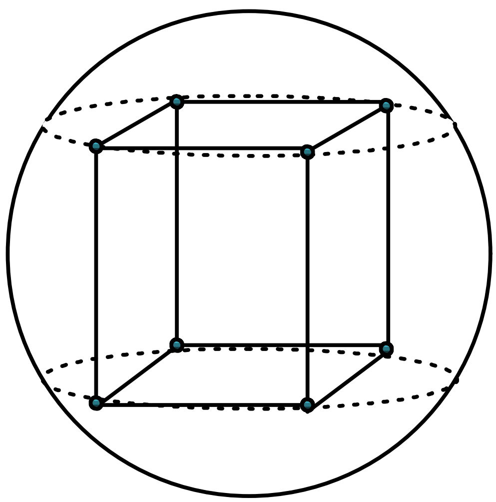
(1) $\frac{MR^2}{16\sqrt{2}\pi}$

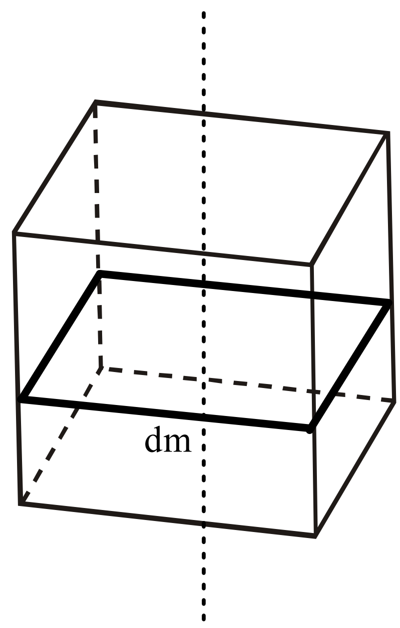
(2) $\frac{4MR^2}{9\sqrt{3}\pi}$

(3) $\frac{4MR^2}{3\sqrt{3}\pi}$

(4) $\frac{MR^2}{32\sqrt{2}\pi}$

Solution





For the cube to have largest volume, the eight vertices of the cube should lie on the surface of the sphere.

Hence, diagonal of the cube = diameter of the sphere

$l\sqrt{3} = 2R$

Mass of cube $= m = \frac{M}{\frac{4}{3}\pi R^3} l^3$

Consider a plate of small thickness and small mass dm.

For this square plate, moment of inertia about an axis passing through its center and perpendicular to its face $= dI = \frac{dml^2}{6}$.

$$I = \int \frac{l^2}{6} dm = \frac{ml^2}{6} = \frac{\left(\frac{M}{\frac{4}{3}\pi R^3}\right) l^3 l^2}{6}$$

$$I = \frac{Ml^5}{8\pi R^3} = \frac{M\left(\frac{2R}{\sqrt{3}}\right)^5}{8\pi R^3} = \frac{4MR^2}{9\sqrt{3}\pi}$$

Hence, Option (2).