

Question

The integral

$$\int_2^4 \frac{\log x^2}{\log x^2 + \log(36 - 12x + x^2)} dx$$

is equal to :

- (1) 4
- (2) 1
- (3) 6
- (4) 2

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Solution

$$I = \int_2^4 \frac{\ln x^2}{\ln x^2 + \ln(36 - 12x + x^2)} dx$$

$$= \int_2^4 \frac{\ln x^2}{\ln x^2 + \ln(6-x)^2} dx$$

$$= \int_2^4 \frac{2 \ln x}{2 \ln x + 2 \ln(6-x)} dx$$

$$I = \int_2^4 \frac{\ln x}{\ln x + \ln(6-x)} dx \dots\dots\dots(*)$$

$$= \int_2^4 \frac{\ln(4+2-x)}{\ln(4+2-x) + \ln\{6-(4+2-x)\}} dx$$

$$I = \int_2^4 \frac{\ln(6-x)}{\ln(6-x) + \ln x} dx \dots\dots\dots(\#)$$

(*)+(#) Gives,

$$2I = \int_2^4 dx$$

$$\therefore I = 1$$

Hence, Option (2).