Let $f(x)$ be a polynomial of degree four having extreme values at $x=1$ and $x=2$.

If $\lim_{x\to 0}\left[1+\frac{f(x)}{x^2}\right]=3$, then $f(2)$ is equal to :

(1) -4

(2) 0

(3) 4

(4) -8

Solution

Let, $f(x)=ax^4+bx^3+cx^2+dx+e$

$$\lim_{x\to 0}\left(1+\frac{ax^4+bx^3+cx^2+dx+e}{x^2}\right)=3$$

$$\Rightarrow \lim_{x\to 0}\frac{ax^4+bx^3+(c+1)x^2+dx+e}{x^2}=3$$

For the above limit to be finite, $d=0$ & $e=0$

$$\therefore \lim_{x\to 0}\frac{ax^4+bx^3+(c+1)x^2}{x^2}=3$$

$$\Rightarrow \lim_{x\to 0}[ax^2+bx+(c+1)]=3$$

$\Rightarrow c+1=3$

$\Rightarrow c=2$

So, $f(x)=ax^4+bx^3+2x^2$

The above function is differentiable and has extremum at 1 & 2.

Hence, $f'(1)=0$ & $f'(2)=0$

Now, $f'(x)=4ax^3+3bx^2+4x$

$f'(1)=4a+3b+4=0$

$f'(2)=32a+12b+8=0$ Or $8a+3b+2=0$

$\therefore a=\frac{1}{2}$ & $b=-2$

$f(2)=\frac{1}{2}\times 2^4+(-2)\times 2^3+2\times 2^2=0$

Hence, (2).