The set of all values of λ for which the system of linear equations :

$2x_1 - 2x_2 + x_3 = \lambda x_1$

$2x_1 - 3x_2 + 2x_3 = \lambda x_2$

$-x_1 + 2x_2 \quad = \lambda x_3$

has a non-trivial solution,

(1) is a singleton.

(2) contains two elements.

(3) contains more than two elements.

(4) is an empty set.

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We have $(2-\lambda)x_1 - 2x_2 + x_3 = 0$, $2x_1 - (3+\lambda)x_2 + 2x_3 = 0$ & $-x_1 + 2x_2 - \lambda x_3 = 0$ system having non-trivial solution.

Hence, $\begin{vmatrix} 2-\lambda & -2 & 1 \\ 2 & -(3+\lambda) & 2 \\ -1 & 2 & -\lambda \end{vmatrix} = 0$

$R_1 \to R_1 + R_3$ yields,

$$\begin{vmatrix} 1-\lambda & 0 & 1-\lambda \\ 2 & -(3+\lambda) & 2 \\ -1 & 2 & -\lambda \end{vmatrix} = 0$$

$$\therefore (1-\lambda)\begin{vmatrix} 1 & 0 & 1 \\ 2 & -(3+\lambda) & 2 \\ -1 & 2 & -\lambda \end{vmatrix} = 0$$

$C_1 \to C_1 - C_3$ yields,

$$(1-\lambda)\begin{vmatrix} 0 & 0 & 1 \\ 0 & -(3+\lambda) & 2 \\ \lambda-1 & 2 & -\lambda \end{vmatrix} = 0$$

Expanding about R_1 yields,

$(1-\lambda)(\lambda-1)(3+\lambda) = 0$

$\therefore \lambda = -3$ or $\lambda = 1$

Hence, Option (2).