Question

The sum of first 9 terms of the series $\frac{1^3}{1}+\frac{1^3+2^3}{1+3}+\frac{1^3+2^3+3^3}{1+3+5}+.....$ is:

(1) 96 (2) 142 (3) 192 (4) 71

Solution

$$t_r=\frac{1^3+2^3+3^3+..... \text{ r terms}}{1+3+5+..... \text{ r terms}}$$

$$\therefore t_r=\frac{\left\{\frac{r(r+1)}{2}\right\}^2}{\frac{r}{2}\{2+(r-1).2\}}$$

$$\Rightarrow t_r=\frac{(r+1)^2}{4}$$

$$S=\sum t_r=\sum\frac{(r+1)^2}{4}=\frac{1}{4}\sum(r+1)^2$$

$$S_9=\frac{1}{4}\sum_{r=1}^{r=9}(r+1)^2$$

$$S_9=\frac{1}{4}(2^2+3^2+4^2+.....+10^2)$$

$$\therefore S_9=\frac{1}{4}\left\{(1^2+2^2+3^2+4^2+.....+10^2)-1\right\}$$

$$\therefore S_9=\frac{1}{4}\left\{\frac{10(10+1)(2\times10+1)}{6}-1\right\}=96$$

Hence, Option (1).