Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three non-zero vectors such that no two of them are collinear and $(\vec{a} \times \vec{b}) \times \vec{c} = \frac{1}{3}|\vec{b}||\vec{c}|\vec{a}$. If θ is the angle between vectors $\vec{b}$ and $\vec{c}$, then a value of $\sin\theta$ is :

(1) $\frac{-\sqrt{2}}{3}$

(2) $\frac{2}{3}$

(3) $\frac{-2\sqrt{3}}{3}$

(4) $\frac{2\sqrt{2}}{3}$

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$$(\vec{a}\times\vec{b})\times\vec{c} = (\vec{a}.\vec{c})\vec{b} - (\vec{b}.\vec{c})\vec{a} = \frac{1}{3}|\vec{b}||\vec{c}|\vec{a}$$

$$\therefore (\vec{a}.\vec{c})\vec{b} = \left\{\frac{1}{3}|\vec{b}||\vec{c}| + (\vec{b}.\vec{c})\right\}\vec{a}$$

Since $\vec{a}$ and $\vec{b}$ are non-collinear vectors,

$(\vec{a}.\vec{c}) = 0$ and $\left\{\frac{1}{3}|\vec{b}||\vec{c}| + (\vec{b}.\vec{c})\right\} = 0$

$$\therefore \vec{b}.\vec{c} = -\frac{1}{3}|\vec{b}||\vec{c}|$$

$$\therefore |\vec{b}||\vec{c}|\cos\theta = -\frac{1}{3}|\vec{b}||\vec{c}|$$

$\therefore \cos\theta = -\frac{1}{3}$ (Since $\vec{b}$ and $\vec{c}$ are non-zero vectors)

$$\therefore \sin\theta = +\sqrt{1-\frac{1}{9}} = \frac{2\sqrt{2}}{3} \quad (\because \quad 0 \le \theta \le \pi)$$

Hence, Option (4).