

If $f(x) + 2f\left(\frac{1}{x}\right) = 3x$, $x \neq 0$, and

$S = \{x \in \mathbf{R} : f(x) = f(-x)\}$; then S :

- (1) is an empty set.
- (2) contains exactly one element.
- (3) contains exactly two elements.
- (4) contains more than two elements.

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We have, $f(x) + 2f\left(\frac{1}{x}\right) = 3x \dots\dots(1)$

Replacing x by $\frac{1}{x}$,

$$f\left(\frac{1}{x}\right) + 2f(x) = \frac{3}{x} \dots\dots(2)$$

(1) + (2) gives,

$$f(x) + f\left(\frac{1}{x}\right) = x + \frac{1}{x} \dots\dots(3)$$

(2) - (1) gives,

$$f(x) - f\left(\frac{1}{x}\right) = \frac{3}{x} - 3x \dots\dots(4)$$

$$\text{From (3) + (4), } 2f(x) = \frac{4}{x} - 2x$$

$$\therefore f(x) = \frac{2}{x} - x$$

$$\therefore f(x) = f(-x), \frac{2}{x} - x = -\frac{2}{x} + x$$

$$\therefore x = \pm\sqrt{2}$$

Hence, Option (3).