The integral $\int \frac{2x^{12}+5x^9}{\left(x^5+x^3+1\right)^3}\,dx$ is equal to :

(1) $\frac{-x^5}{\left(x^5+x^3+1\right)^2}+C$

(2) $\frac{x^{10}}{2\left(x^5+x^3+1\right)^2}+C$

(3) $\frac{x^5}{2\left(x^5+x^3+1\right)^2}+C$

(4) $\frac{-x^{10}}{2\left(x^5+x^3+1\right)^2}+C$

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$$I=\int \frac{2x^{12}+5x^9}{(x^5+x^3+1)^3}dx$$

The highest power on x in the denominator after cubic expansion is 15. So, dividing numerator and denominator by x^{15},

$$I=\int \frac{\frac{2}{x^3}+\frac{5}{x^6}}{\left(1+\frac{1}{x^2}+\frac{1}{x^5}\right)^3}dx$$

The expression inside the integral is of the form $-\frac{f'(x)}{f^3(x)}$, where $f(x)=1+\frac{1}{x^2}+\frac{1}{x^5}$.

$$\therefore I=-\left\{\frac{f^{-3+1}(x)}{-3+1}\right\}+C=\frac{1}{2f^2(x)}+C$$

$$\therefore I=\frac{1}{2\left(1+\frac{1}{x^2}+\frac{1}{x^5}\right)^2}+C$$

$$\therefore I=\frac{x^{10}}{2(x^5+x^3+1)^2}+C$$

Hence, Option (2).