

The system of linear equations

$$x + \lambda y - z = 0$$

$$\lambda x - y - z = 0$$

$$x + y - \lambda z = 0$$

has a non-trivial solution for:

(1) infinitely many values of  $\lambda$ .

(2) exactly one value of  $\lambda$ .

(3) exactly two values of  $\lambda$ .

(4) exactly three values of  $\lambda$ .

Based on JEE Main 2016

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For non-trivial solution, 
$$\begin{vmatrix} 1 & \lambda & -1 \\ \lambda & -1 & -1 \\ 1 & 1 & -\lambda \end{vmatrix} = 0$$

$$\therefore \begin{vmatrix} 1 & \lambda+1 & -1 \\ \lambda & 0 & -1 \\ 1 & 1+\lambda & -\lambda \end{vmatrix}_{C_2 \rightarrow C_2 - C_3} = 0$$

$$\therefore (\lambda+1) \begin{vmatrix} 1 & 1 & -1 \\ \lambda & 0 & -1 \\ 1 & 1 & -\lambda \end{vmatrix} = 0$$

$$\therefore (\lambda+1) \begin{vmatrix} 0 & 0 & \lambda-1 \\ \lambda & 0 & -1 \\ 1 & 1 & -\lambda \end{vmatrix}_{R_1 \rightarrow R_1 - R_3} = 0$$

$$\therefore (\lambda+1)(\lambda-1) \begin{vmatrix} \lambda & 0 \\ 1 & 1 \end{vmatrix}_{\text{Expanding about } R_1} = 0$$

$$\therefore (\lambda+1)(\lambda-1)\lambda = 0$$

$$\therefore \lambda = -1, 0, 1$$

Hence, Option (4).