Consider ΔABC in which D is the mid-point of side BC. If $AD \perp AC$,

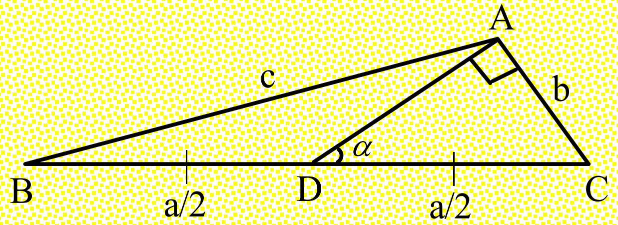
A) $\cos A.\cos C = \dfrac{2(a^2 - c^2)}{3ac}$

C) $\cos A.\cos C = \dfrac{2(a^2 - c^2)}{3b^2}$

B) $\cos A.\cos C = \dfrac{2(c^2 - a^2)}{3ac}$

D) $\cos A.\cos C = \dfrac{2(c^2 - a^2)}{3b^2}$

Solution



$$\cos A.\cos C = \frac{b^2 + c^2 - a^2}{2bc}.\frac{a^2 + b^2 - c^2}{2ab}$$

From $\Delta ACD, AD^2 + b^2 = \left(\frac{a}{2}\right)^2$ ....(*) and $\cos\alpha = \dfrac{AD}{a/2}$.......(#)

From $\Delta ABD, \cos(180 - \alpha) = \dfrac{AD^2 + \left(\frac{a}{2}\right)^2 - c^2}{2.\frac{a}{2}.AD}$

$$\therefore c^2 = AD^2 + \left(\frac{a}{2}\right)^2 + a.AD.\cos\alpha$$

$$\therefore c^2 = AD^2 + \left(\frac{a}{2}\right)^2 + a.AD.\frac{2AD}{a} \qquad \text{[From \#]}$$

$$\therefore c^2 = 3AD^2 + \left(\frac{a}{2}\right)^2$$

$$\therefore c^2 = 3\left\{\left(\frac{a}{2}\right)^2 - b^2\right\} + \left(\frac{a}{2}\right)^2 \qquad \text{[From *]}$$

$$\therefore c^2 = 4\left(\frac{a}{2}\right)^2 - 3b^2 = a^2 - 3b^2$$

$$\therefore b^2 = \frac{a^2 - c^2}{3}$$

Now, $\cos A.\cos C = \dfrac{b^2 - 3b^2}{2bc}.\dfrac{3b^2 + b^2}{2ab} = \dfrac{-2b^2}{ac} = \dfrac{2(c^2 - a^2)}{3ac}$

Hence, Option (B).