

Evaluate, $\int \frac{\sin x}{\cos 3x} + \frac{\sin 3x}{\cos 9x} + \frac{\sin 9x}{\cos 27x} dx$

Solution

The given integral,

$$\begin{aligned} &= \int \frac{\sin 2x}{2\cos 3x \cos x} + \frac{\sin 6x}{2\cos 9x \cos 3x} + \frac{\sin 18x}{2\cos 27x \cos 9x} dx \\ &= \int \frac{\sin(3x-x)}{2\cos 3x \cos x} + \frac{\sin(9x-3x)}{2\cos 9x \cos 3x} + \frac{\sin(27x-9x)}{2\cos 27x \cos 9x} dx \\ &= \int \frac{\sin 3x \cos x - \cos 3x \sin x}{2\cos 3x \cos x} + \frac{\sin 9x \cos 3x - \cos 9x \sin 3x}{2\cos 9x \cos 3x} + \frac{\sin 27x \cos 9x - \cos 27x \sin 9x}{2\cos 27x \cos 9x} dx \\ &= \int \frac{1}{2} (\tan 3x - \tan x + \tan 9x - \tan 3x + \tan 27x - \tan 9x) dx \\ &= \int \frac{1}{2} (-\tan x + \tan 27x) dx \\ &= \frac{1}{2} \ln |\cos x| - \frac{\ln |\cos 27x|}{54} + C \end{aligned}$$